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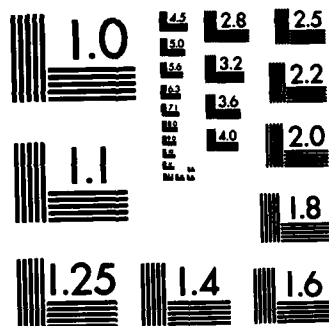
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CONVERSION OF EXISTING CONTINUOUS CONTROL SYSTEMS INTO DIGITAL CONTROL SYSTEMS

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ABSTRACT

The problem of converting existing continuous-data control systems into digital control systems is considered. The objective of this paper is to develop a computer-aided method for synthesizing the pulse-transfer function of the digital controller. This is done by matching the frequency response of the digital control system to that of the continuous system with a minimum weighted mean square error. Formulas for computing the parameters of the digital controller are obtained as a result. The design technique is illustrated with a numerical example and a comparison with previous methods is also presented.

INTRODUCTION

Digital controls are the subject of expanding interest in both the academic and industrial communities. This is especially true since the advent of the economical microprocessor and its related support chips which have made digital controls an attractive alternative when considering a control strategy. For the past few years new control systems have been designed using digital instead of analog controllers. There is also a need for converting systems which were designed to be controlled by continuous controllers into systems to be controlled by digital controllers. Conversion of existing control systems is a problem of considerable interest to various branches of industry.

A popular method for converting existing continuous control systems into digital control systems is by Tabak¹. This method converts the transfer function of the continuous controller into the z-transfer function of the digital controller using prewarped bilinear transformation. The method is straightforward and easy to use. But the digital control system obtained by this method approximates the continuous control system only when the sampling frequency is sufficiently high as compared with the highest frequency of the continuous system. This is because the transfer function of the plant and the feedback are not taken into consideration. Thus the capabilities of the digital controller are not fully utilized.

Recently the author of this paper and Yeh² proposed a computer-aided method for synthesizing a digital controller by matching the "discrete

frequency response" of the digital control system to the frequency response of the continuous model with a minimum weighted mean square error. The "discrete frequency response" of the digital control system is obtained by putting an artificial sampler of duration T at the output of the system and then substituting $z = e^{j\omega T}$ in the overall pulse-transfer function of the system. Even though the results obtained by this method were superior to the ones obtained by Tabak's method¹, this method only matched the response of the systems at the sampling instants. The purpose of this study is to improve on this method by matching the "continuous frequency response" of the digital control system with that of the continuous model. The "continuous frequency responses" of the control systems are obtained by putting an artificial sampler of duration $T_s=0$ at the output and input of the systems, substituting in the overall

transfer functions $z = \frac{1 + \frac{j\omega T}{2}}{1 - \frac{j\omega T}{2}}$, $s = j\omega$; and then

substituting $w = j\gamma$ (where γ is the frequency in the w-domain). The synthesis of the digital controller is carried out by minimizing an error between the "continuous frequency responses" of the digital and existing systems in the w-domain. As a result the formulas for computing the parameters of the digital controller are obtained.

DIGITAL CONTROLLER DESIGN
 VIA CONTINUOUS FREQUENCY MATCHING

The block diagram of an existing continuous-data control system may be drawn as shown in Fig. 1.

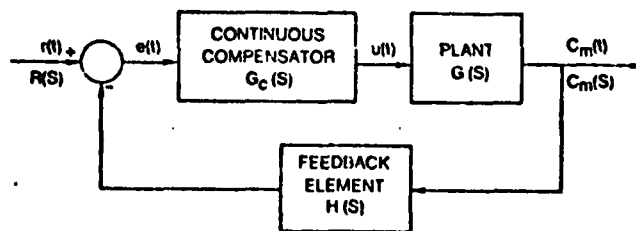


Fig. 1. Existing continuous-data control system

The continuous model shown in Fig. 1 can be digi-

talized by inserting a sampler of duration T at the error input $e(t)$ and replacing the continuous compensator $G_c(s)$ by a digital controller $D(z)$ and a zero-order hold as shown in Fig. 2. The sampling frequency $\omega_s = \frac{2\pi}{T}$ is selected to be sufficiently high as compared to the highest frequency of the continuous system.

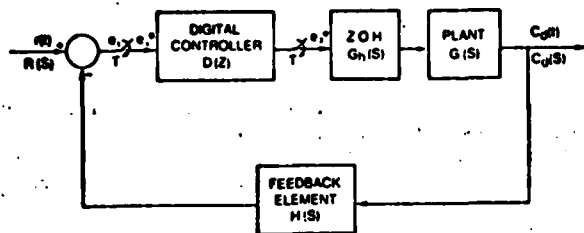


Fig. 2. Redesigned digital control system

The overall transfer function of the continuous and digital systems can be written from Figs. 1 and 2 respectively as

$$\frac{C_m(s)}{R(s)} = F(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} \quad (1)$$

$$\frac{C_d(s)}{R^T(s)} = G_d(s) = \frac{D^T(s)G_h G(s)}{1 + D^T(s)G_h G^T(s)} \quad (2)$$

where $G_c(s)$, $G_h(s)$, $G(s)$ and $H(s)$ are the transfer functions of the continuous controller, zero-order hold, plant and the feedback element respectively. The superscript notation denotes the sampling rate. For example, R^T indicates that the signal R is sampled at $\frac{1}{T}$ samples/second.

It is required to design a digital controller so that the steady state output of the digital control system follows the desired output of the continuous model for all sinusoidal inputs within the frequency range. This is done by matching the frequency response of the digital control system to the ideal frequency response of the continuous system.

The frequency response of the continuous model can be obtained by substituting $s = j\omega$ in eqn. (1) and varying ω from 0 to ∞ . But because of the hybrid nature of eqn. (2), a common way of finding the frequency response of the digital control system is to put an artificial sampler of sampling duration T secs. at the output of the system, substitute $z = e^{j\omega T}$ in the overall z -transfer function and then vary ω from 0 to π/T . We will call this "discrete-frequency response" or "sampled spectrum". Recently the Author and Yeh² proposed a method for synthesizing the pulse-transfer function of the digital controller by matching the discrete response of the digital control system to the frequency

response of the continuous model. In this method we only matched the response at the sampling instants. It is this observation which prompted the development of the method in this paper which matches the "continuous frequency response" of both systems. To do this we put an artificial sampler of duration $T_1 \rightarrow 0$ secs at the output and input of the systems shown in Figures 1 and 2.

Thus the overall transfer functions of the systems shown in Figs. 1 and 2 with a sampler $T_1 \rightarrow 0$ at the input and output can be written as

$$\frac{C_m^{T_1}(s)}{R^T(s)} = F^{T_1}(s) \quad (3)$$

$$\frac{C_d^{T_1}(s)}{R^T(s)} = G_d^{T_1}(s) = \frac{D^T(s)G_h G^{T_1}(s)}{1 + D^T(s)G_h G^T(s)} \quad (4)$$

The "continuous frequency response" of the continuous and digital control system can now be obtained by transforming eqns. (3) and (4) into w domain which is related to the z -domain by a bilinear transformation ($w = \frac{2}{T} \frac{z-1}{z+1}$) and the s -domain by $w \rightarrow s$ as $T_1 \rightarrow 0$ (Whitback and Hofmann³) and then substituting $w = j\gamma$ in the overall transfer functions. As $T_1 \rightarrow 0$, the overall transfer functions of the continuous and digital control systems given by eqns. (3) and (4) can be written in the w -domain as

$$F(s) = F(s) \Big|_{s=w} \quad (5)$$

$$G_d(w) = \frac{D(z)G_h G(s)}{1 + D(z)G_h G^T(z)} \Big|_{z = \frac{1 + \frac{wT}{2}}{1 - \frac{wT}{2}}, s=w} \quad (6)$$

Equation (6) can be written as

$$G_d(w) = \frac{D(z)G(w)}{1 + D(z)G_h G^T(z)} \Big|_{z = \frac{1 + \frac{wT}{2}}{1 - \frac{wT}{2}}} \quad (7)$$

Let the transfer function of the digital controller in the w -domain be written as

$$D(w) = \frac{a_m w^m + a_{m-1} w^{m-1} + \dots + a_1 w + a_0}{b_n w^n + b_{n-1} w^{n-1} + \dots + b_1 w + 1}, \quad m \leq n \quad (8)$$

where a's and b's are constants to be determined. Substituting $w = jy$ in eqns. (8) and (7), we get

$$D(jy) = \frac{P+jyQ}{1+L+jyM} \quad (9)$$

where

$$P = a_0 - a_2y^2 + a_4y^4 - a_6y^6 + \dots$$

$$Q = a_1 - a_3y^2 + a_5y^4 - a_7y^6 + \dots$$

$$L = -b_2y^2 + b_4y^4 - b_6y^6 + \dots$$

$$M = b_1 - b_3y^2 + b_5y^4 - b_7y^6 + \dots$$

and

$$G_d(jy) = \frac{\frac{P+jyQ}{1+L+jyM} (X_1+jY_1)}{1 + \frac{P+jyQ}{1+L+jyM} (X_2+jY_2)} \quad (10)$$

where

$$G_h G(jy) = G_h G(s) \Big|_{s=jy} = X_1+jY_1$$

$$G_h GH(jy) = G_h GH(z) \Big|_{z = \frac{1+jyT/2}{1-jyT/2}} = X_2+jY_2$$

For notational convenience, let the numerator and denominator of $G_d(jy)$ be denoted by $N(y)$ and $M(y)$ respectively, i.e.

$$G_d(jy) = \frac{N(y)}{M(y)} \quad (11)$$

In terms of real and imaginary parts, $N(y)$ and $M(y)$ can be written as

$$N(y) = \phi + j\theta \quad (12)$$

$$M(y) = \sigma + j\tau \quad (13)$$

where

$$\phi = PX_1 - YQY_1$$

$$\theta = PY_1 + YQX_1$$

$$\sigma = 1 + L + PX_2 - YQY_2$$

$$\tau = YH + PY_2 + YQX_2$$

The transfer function $F(jy)$ of eqn. (5) can also be written in terms of real and imaginary part as

$$F(jy) = A + jB \quad (14)$$

The error between $G_d(jy)$ and $F(jy)$ is defined as

$$e(y) = F(jy) - G_d(jy) \quad (15)$$

An optimal set of a's and b's can therefore be obtained by minimizing the mean square of the magnitude of $e(y)$. Let the error function to be minimized be defined as

$$E = \int_{y_1}^{y_2} \left| F(jy) - \frac{N(y)}{M(y)} \right|^2 dy \quad (16)$$

where $(y_2 - y_1)$ is the bandwidth in the w -domain on which the matching is required. This error function is to be minimized by adjusting the unknown coefficients a_i and b_i . However, due to the presence of $M(y)$ in the denominator, the equations obtained by differentiating E with respect to a_i and b_i are nonlinear. This nonlinearity may be removed by modifying the error function (16) as

$$E_m = \int_{y_1}^{y_2} |F(jy)M(y) - N(y)|^2 dy \quad (17)$$

This integral differs from that in eqn. (16) by including $|M(y)|^2$ in the integrand as a weighting factor. This modification is not new. It was suggested by Kalman⁴ as an identification technique, was used by Levy⁵ in complex curve fitting, Rao and Lamba⁶ for simplifying linear systems and by the author and Yeh² for discretizing continuous systems.

The integrand in eqn. (17) is a complex function which may be separated into real and imaginary part as

$$|F(jy)M(y) - N(y)| = l(y) + jm(y) \quad (18)$$

where

$$l(y) = A\sigma - B\tau - \phi \quad (19)$$

$$m(y) = A\tau + B\sigma - \theta \quad (20)$$

By substituting eqns. (18)-(20) in (17), we get

$$E_m = \int_{y_1}^{y_2} \left[(A\sigma - B\tau - \phi)^2 + (A\tau + B\sigma - \theta)^2 \right] dy \quad (21)$$

Differentiating E_m with respect to a's and b's and equating the resulting equations equal to zero represents $(m+n+1)$ linear equations in $(m+n+1)$ unknowns a_0 to a_m and b_1 to b_n . These equations can be written in the matrix form as given in eqn. (22)

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} T_0 & 0 & -T_2 & 0 & T_4 & 0 & \dots & v_1 & -u_2 & -v_3 & u_4 & v_5 & -u_6 & \dots & \dots \\ 0 & T_2 & 0 & -T_4 & 0 & T_6 & \dots & u_2 & v_3 & -u_4 & -v_5 & u_6 & -v_7 & \dots & \dots \\ -T_2 & 0 & T_4 & 0 & -T_6 & 0 & \dots & -v_3 & u_4 & v_5 & -u_6 & -v_7 & u_8 & \dots & \dots \\ 0 & -T_4 & 0 & T_6 & 0 & -T_8 & \dots & -u_4 & -v_5 & u_6 & v_7 & -u_8 & -v_8 & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_1 & u_2 & -v_3 & -u_4 & v_5 & u_6 & \dots & w_2 & 0 & -w_4 & 0 & w_6 & 0 & \dots & \dots \\ -u_2 & v_3 & u_4 & -v_5 & -u_6 & v_7 & \dots & 0 & w_4 & 0 & -w_6 & 0 & w_8 & \dots & \dots \\ -v_3 & -u_4 & v_5 & u_6 & -v_7 & -u_8 & \dots & -w_4 & 0 & w_6 & 0 & -w_8 & 0 & \dots & \dots \\ u_4 & -v_5 & -u_6 & v_7 & u_8 & -v_9 & \dots & 0 & w_6 & 0 & w_8 & 0 & -w_{10} & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}^{-1} \begin{bmatrix} -u_0 \\ v_1 \\ u_2 \\ -v_3 \\ \vdots \\ 0 \\ w_2 \\ 0 \\ w_4 \\ \vdots \end{bmatrix} \quad (22)$$

where

$$T_h = \int_1^2 \left[(A^2+B^2) (X_2^2+Y_2^2) + (X_1^2+Y_1^2) + 2B(X_1Y_2-X_2Y_1) - 2A(X_1X_2+Y_1Y_2) \right] \gamma^h d\gamma \quad (23)$$

$$u_h = \int_1^2 (A^2+B^2) X^2 - (AX_1+BY_1) \gamma^h d\gamma \quad (24)$$

$$v_h = \int_1^2 (A^2+B^2) Y^2 + (BX_1-AY_1) \gamma^h d\gamma \quad (25)$$

$$w_h = \int_1^2 (A^2+B^2) \gamma^h d\gamma \quad (26)$$

Eqs. (22)-(26) provide a design algorithm which has been programmed on a digital computer, IBM (370/3031) and is available in the form of a subroutine written in FORTRAN IV. Note that the order of the digital controller (integer m and n) is arbitrary. Therefore, using this algorithm, one may try out several digital controllers of different orders and select the best design by considering the tradeoff between cost (complexity) and performance.

NUMERICAL EXAMPLE

The example considered in this paper is to discretize the existing continuous control system shown in Fig. 1. The transfer function of the continuous controller, plant and the feedback element is given by

$$G_c(s) = \frac{1+0.416s}{1+0.139s} \quad (27)$$

$$G(s) = \frac{10}{s(s+1)} \quad (28)$$

$$H(s) = 1 \quad (29)$$

Using eqns. (27)-(29), the transfer function of the continuous model can be written as

$$F(s) = \frac{29.93s + 71.942}{s^3 + 8.194s^2 + 37.12s + 71.942} \quad (30)$$

The magnitude of the frequency response of the continuous model is obtained using eqn. (30) and is shown in Fig. 3. Based on the frequency response, the sampling period T is selected to be 0.15s. The transfer function of the zero-order hold is given by

$$G_h(s) = \frac{1-e^{-sT}}{s} \quad (31)$$

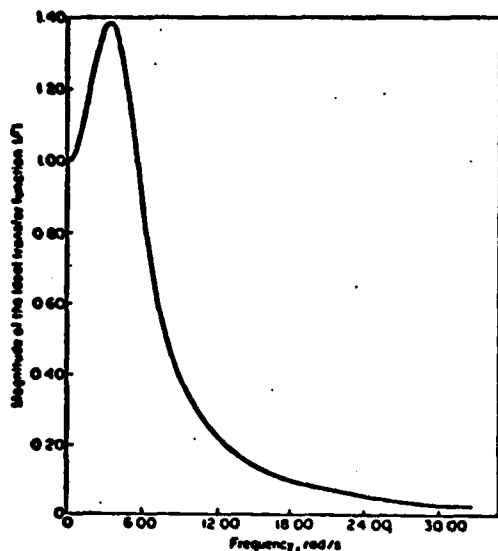


Fig. 3. Magnitude of the frequency response of the continuous system

From eqns. (28), (29) and (31),

$$G_h GH(s) = \frac{10(1-e^{-sT})}{s^2(s+1)} \quad (32)$$

The z-transform associated with this transfer function is given by

$$G_h GH(z) = \frac{0.10708z + 0.101858}{z^2 - 1.86072z + 0.8607} \quad (33)$$

Substituting $z = \frac{1+j\omega T/2}{1-j\omega T/2}$ in eqn. (33), we get

$$G_h GH(j\omega) = X_2 + jY_2 = \frac{9.981 - .73w - .0019w^2}{w(w+.998)} \quad (34)$$

$w = j\omega$

Substituting $s = j\omega$ in eqns (28) and (30), we get

$$G(j\omega) = X_1 + jY_1 = \frac{10}{w(w+1)} \quad (35)$$

$w = j\omega$

$$F(j\omega) = A + jB = \frac{29.93w + 71.942}{w^3 + 8.194w^2 + 37.142w + 71.942} \quad (36)$$

$w = j\omega$

Eqns. (30), (34) - (36) are the only information required to design a digital controller for the digital control system shown in Fig. 2. Feeding these eqns. to the computer program, the transfer function of the first-order digital controller given by the algorithm in the w-domain is

$$D(w) = \frac{1.00516 + .34064w}{1 + .04606w} \quad (37)$$

Substituting $w = \frac{2}{T} \frac{z-1}{z+1}$ in eqn. (37) gives the pulse-transfer function of the digital controller

$$D(z) = \frac{3.436z - 2.191}{z + .2390} \quad (38)$$

The frequency responses of the existing continuous system and the digital control system with the digital controller given by eqn. (38) are shown in Fig. 4-6. It can be seen that both the magnitude and phase angle of the digital system matches closely with that of the continuous model. If the designer is not satisfied with the results and wants to improve the performance, the only change which needs to be made is to increase the order of the digital controller. The algorithm goes through the standard steps given above and finds the parameters of the new digital controller. Thus the performance may be improved at the expense of increasing complexity.

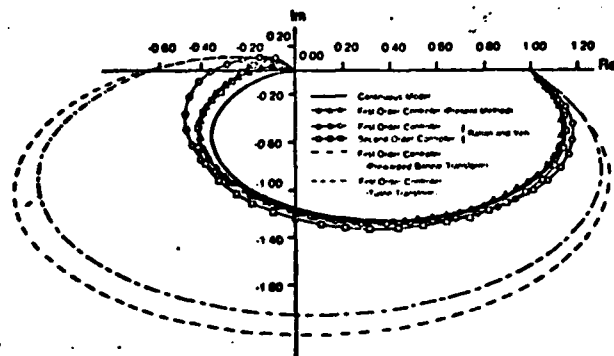


Figure 4. Nyquist plot of the frequency responses of the continuous system and the digital control system designed by different methods.

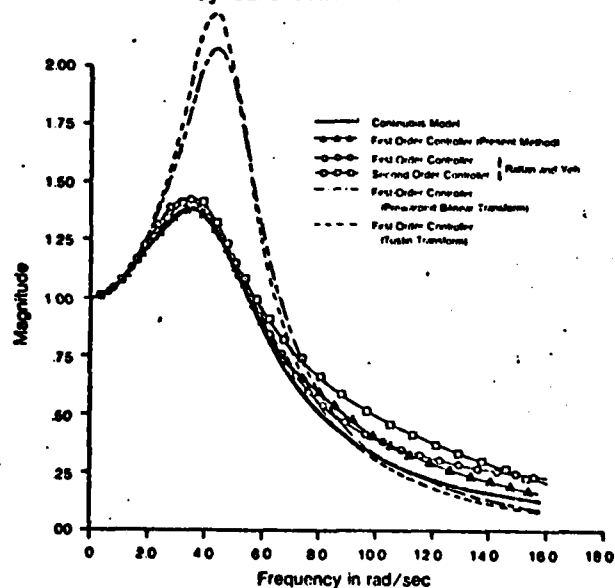


Figure 5. Magnitude plot of the frequency responses of the continuous system and the digital control system designed by different methods.

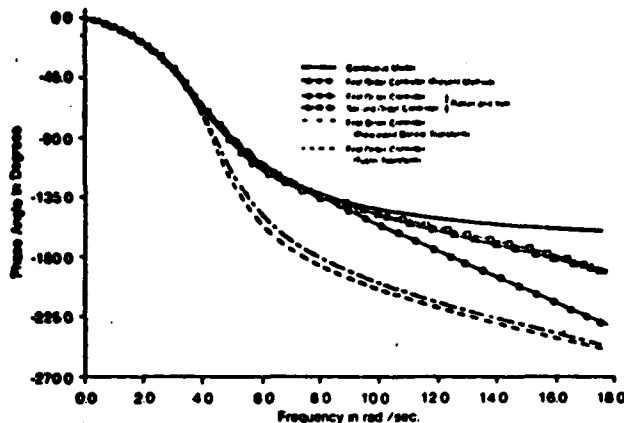


Figure 6. Phase plot of the frequency responses of the continuous system and the digital control system designed by different methods.

COMPARISON WITH THE EXISTING METHODS

The digital controller for the digital control system discussed in the numerical example is re-designed using the Tustin transform⁷ (bilinear transformation); methods presented by Tabak¹ (pre-warped bilinear transformation) and by the author of the paper and Yeh² (discrete frequency matching). Using these methods, the continuous controller given in eqn. (27) is converted into digital controllers and are given by

Tustin transform:

$$D(z) = \frac{2.294z - 1.5936}{z - 0.299} \quad (39)$$

Prewarped bilinear transform:

$$D(z) = \frac{2.42965z - 1.68046}{z - 0.2509} \quad (40)$$

Using the method presented by the author and Yeh²:

first order:

$$D(z) = \frac{3.6787z - 2.31256}{z + 0.383656} \quad (41)$$

second order:

$$D(z) = \frac{3.96z^2 - 3.65z + 0.56}{z^2 - 0.34z - 0.384} \quad (42)$$

The frequency responses of the digital control system with digital controllers given by eqns. (39)-(42) are shown in Figs. 4-6. Comparison of the results in these figures demonstrate that the digital controller obtained by the present method gives better results.

CONCLUSIONS

A method for converting an existing continuous control system into a digital control system with similar performances is developed. An attempt is made to match the "continuous frequency response" of the digital control system as closely as

possible with that of the continuous model. The parameters of the digital controller are obtained by minimizing a modified mean-square of the error between the transfer function of the continuous and the digital systems in the w -domain. An explicit and straightforward formula for computing the parameters is obtained and has been programmed for a computer. Since the design procedure is computer-aided, one may try out several digital controllers of different order and select the best design by considering the trade-off between the cost and performance.

The numerical example considered shows that the performance of the digital control system obtained by the method presented is superior to the ones obtained by the existing methods.

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